

<p>1(i) $x = 1$</p>	<p>B1 [1]</p>	
<p>(ii) $\frac{dy}{dx} = \frac{(x-1)2x - (x^2+3) \cdot 1}{(x-1)^2}$ $= \frac{2x^2 - 2x - x^2 - 3}{(x-1)^2}$ $= \frac{x^2 - 2x - 3}{(x-1)^2}$ $dy/dx = 0$ when $x^2 - 2x - 3 = 0$ $\Rightarrow (x-3)(x+1) = 0$ $\Rightarrow x = 3$ or -1 When $x = 3$, $y = (9+3)/2 = 6$ So P is (3, 6)</p>	<p>M1 A1 M1 M1 A1 B1ft [6]</p>	<p>Quotient rule correct expression their numerator = 0 solving quadratic by any valid method $x = 3$ from correct working $y = 6$</p>
<p>(iii) Area = $\int_2^3 \frac{x^2+3}{x-1} dx$ $u = x - 1 \Rightarrow du/dx = 1, du = dx$ When $x = 2, u = 1$; when $x = 3, u = 2$ $= \int_1^2 \frac{(u+1)^2+3}{u} du$ $= \int_1^2 \frac{u^2+2u+4}{u} du$ $= \int_1^2 (u+2+\frac{4}{u}) du$ * $= \left[\frac{1}{2}u^2 + 2u + 4\ln u \right]_1^2$ $= (2 + 4 + 4\ln 2) - (\frac{1}{2} + 2 + 4\ln 1)$ $= 3\frac{1}{2} + 4\ln 2$</p>	<p>M1 B1 B1 E1 B1 M1 A1cao [7]</p>	<p>Correct integral and limits Limits changed, and substituting $dx = du$ substituting $\frac{(u+1)^2+3}{u}$ www [$\frac{1}{2} u^2 + 2u + 4\ln u$] substituting correct limits</p>
<p>(iv) $e^y = \frac{x^2+3}{x-1}$ $\Rightarrow e^y \frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2}$ $\Rightarrow \frac{dy}{dx} = e^{-y} \frac{x^2-2x-3}{(x-1)^2}$ When $x = 2, e^y = 7 \Rightarrow$ $\Rightarrow dy/dx = \frac{1}{7} \cdot \frac{4-4-3}{1} = -\frac{3}{7}$</p>	<p>M1 A1ft B1 A1cao [4]</p>	<p>$e^y dy/dx =$ their $f'(x)$ or $xe^y - e^y = x^2 + 3$ $\Rightarrow e^y + xe^y \frac{dy}{dx} - e^y \frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x - e^y}{e^y(x-1)}$ $y = \ln 7$ or $1.95\dots$ or $e^y = 7$ or $\frac{dy}{dx} = \frac{4-7}{7(2-1)} = -\frac{3}{7}$ or -0.43 or better</p>

<p>2 $y^2 + y = x^3 + 2x$ $x = 2 \Rightarrow y^2 + y = 12$ $\Rightarrow y^2 + y - 12 = 0$ $\Rightarrow (y - 3)(y + 4) = 0$ $\Rightarrow y = 3 \text{ or } -4.$ $2y \frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx}(2y + 1) = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx} = \frac{3x^2 + 2}{2y + 1}$ At (2, 3), $\frac{dy}{dx} = \frac{12 + 2}{6 + 1} = 2$ At (2, -4), $\frac{dy}{dx} = \frac{12 + 2}{-8 + 1} = -2$</p>	<p>M1 A1 A1 M1 A1cao M1 A1 cao A1 cao [8]</p>	<p>Substituting $x = 2$ $y = 3$ $y = -4$ Implicit differentiation – LHS must be correct substituting $x = 2, y = 3$ into their dy/dx, but must require both x and one of their y to be substituted 2 -2</p>
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Question		Answer	Marks	Guidance
3	(i)	$f(-x) = \frac{-x}{\sqrt{2+(-x)^2}}$ $= -\frac{x}{\sqrt{2+x^2}} = -f(x)$ <p>Rotational symmetry of order 2 about O</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>[3]</p>	<p>substituting $-x$ for x in $f(x)$</p> <p>1st line must be shown, must have $f(-x) = -f(x)$ oe somewhere</p> <p>must have 'rotate' and 'O' and 'order 2 or 180 or 1/2 turn'</p> <p>$\frac{-x}{\sqrt{2+(-x)^2}}, \frac{-x}{\sqrt{2+-(x^2)}}, \frac{-x}{\sqrt{2+(-x^2)}}$ M1A0</p> <p>$\frac{-x}{\sqrt{2-x^2}}$ M0A0</p> <p>oe e.g. reflections in both x- and y-axes</p>
	(ii)	$f'(x) = \frac{\sqrt{2+x^2} \cdot 1 - x \cdot \frac{1}{2}(2+x^2)^{-1/2} \cdot 2x}{(\sqrt{2+x^2})^2}$ $= \frac{2+x^2-x^2}{(2+x^2)^{3/2}} = \frac{2}{(2+x^2)^{3/2}} *$ <p>When $x = 0, f'(x) = 2/2^{3/2} = 1/\sqrt{2}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>[5]</p>	<p>quotient or product rule used</p> <p>$\frac{1}{2} u^{-1/2}$ or $-\frac{1}{2} v^{-3/2}$ soi</p> <p>correct expression</p> <p>NB AG</p> <p>oe e.g. $\sqrt{2}/2, 2^{-1/2}, 1/2^{1/2}$, but not $2/2^{3/2}$</p> <p>QR: condone $udv \pm vdu$, but u, v and denom must be correct</p> <p>$x(-1/2)(2+x^2)^{-3/2} \cdot 2x + (2+x^2)^{-1/2}$</p> <p>$= (2+x^2)^{-3/2}(-x^2 + 2 + x^2)$</p> <p>allow isw on these seen</p>
	(iii)	$A = \int_0^1 \frac{x}{\sqrt{2+x^2}} [dx]$ <p>let $u = 2 + x^2, du = 2x dx$</p> $= \int_2^3 \frac{1}{2} \frac{1}{\sqrt{u}} du$ $= \left[u^{1/2} \right]_2^3$ $= \sqrt{3} - \sqrt{2}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1cao</p> <p>[4]</p>	<p>correct integral and limits</p> <p>or $v = \sqrt{2+x^2}, dv = x(2+x^2)^{-1/2} dx$</p> <p>$\int \frac{1}{2} \frac{1}{\sqrt{u}} [du]$ or $\int 1 [dv]$ or $k(2+x^2)^{1/2}$</p> <p>$[u^{1/2}]$ o.e. (but not $1/u^{-1/2}$) or $[v]$ or $k = 1$</p> <p>must be exact</p> <p>limits may be inferred from subsequent working, condone no dx</p> <p>condone no du or dv, but not $\int \frac{1}{2} \frac{1}{\sqrt{u}} dx$</p> <p>isw approximations</p>

Question			Answer	Marks	Guidance
3	(iv)	(A)	$y^2 = \frac{x^2}{2+x^2}$ $\Rightarrow 1/y^2 = (2+x^2)/x^2 = 2/x^2 + 1 *$	M1 A1 [2]	squaring (correctly) or equivalent algebra NB AG must show $\left[\sqrt{(2+x^2)}\right]^2 + 2+x^2$ (o.e.) If argued backwards from given result without error, SCB1
	(iv)	(B)	$-2y^{-3}dy/dx = -4x^{-3}$ $\Rightarrow dy/dx = -4x^{-3}/-2y^{-3} = 2y^3/x^3 *$ <p>Not possible to substitute $x = 0$ and $y = 0$ into this expression</p>	B1B1 B1 B1 [4]	LHS, RHS NB AG soi (e.g. mention of 0/0) condone $dy/dx -2y^{-3}$ unless pursued Condone 'can't substitute $x = 0$ ' o.e. (i.e. need not mention $y = 0$). Condone also 'division by 0 is infinite'